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Fuzzy Expert Program for Dosage Adjusting in Medical Treatment Comparative Study of Different Fuzzy Logic Steps.

Content

1 Introduction.

In the last time the Artificial intelligence has been used to solve many complex problems, by developing intelligent systems.
Fuzzy logic has been proved to be a powerful tool for decision making systems such as expert systems. Fuzzy set theory has already been used in some medical expert systems.
Uncertainty is the crucial, critical fact about medical reasoning. Patients cannot describe exactly what has happened to them or how they fill, doctors cannot tell exactly what they observe, laboratory report results only with some degree of error, physiologists do not understand precisely how the human body works, medical researchers cannot precisely describe how diseases alter the normal functioning of the body, pharmacologists do not fully understand the mechanism accounting for the effectiveness of drugs, and no one can precisely determine one' s prognosis.
Nevertheless we must make important, even crucial decisions testing and treatments and, despite the uncertainties about the bases for those decisions, the decision themselves must be definitive (P.Szolovitz Uncertainty and Decisions in Medical Informatics. Methods of Information in Medicine 34 , 1995 *)*
The medical knowledge concerning the symptom – disease relationship constitutes one source of imprecision and uncertainty in the diagnosis process, and the knowing of the state of the patient constitutes another.
The physicians generally gather knowledge about the patient from the past history, physical examination, laboratory results and other investigative procedures such as X –rays and ultrasound studies. The information provided by each of these sources carries with it varying degrees of uncertainty. The past history offered by the patient may be subjective, exaggerated, underestimated or incomplete. Mistakes may be made in physical examination and symptoms may be overlooked. The measurements provided by the laboratory tests are

often of limited precision, and the exact borderline between normal and pathological is often unclear. Thus the physician can know only with a limited degree of precision the state and symptoms of the patient. In the face of uncertainty concerning the symptoms to a disease entity it is nevertheless crucial that the physician determines the diagnostic label that will entail the appropriate therapeutic regimen. In order to understand better this difficult and important process of medical diagnosis and treatment, it can be modeled with the use of fuzzy sets. These models vary in the degree to which they attempt deal with different implicating aspects of medical diagnosis**.**

2.Basic concepts of fuzzy sets and fuzzy inference.

Medical fuzziness is imprecision: a fuzzy proposition may be true in *some degree*. The word crisp is used as meaning " *non fuzzy* ". Standard examples of fuzzy propositions use linguistic variables as age with possible values: young, medium old or similar. The sentence " the patient is old " is true in some degree, the bigger is the age the more the true.
Truth of fuzzy proposition is a matter of degree.

2.1 Fuzzy sets.

Fuzzy set is a set with imprecise boundaries in which the transition from membership to non-membership is gradual rather than abrupt.

Univers*e*. Elements of a fuzzy set are taken from a universe of discourse, or universe for short.The univers contains all elements that can come into consideration.

Membership function*.*Every element in the universe of discours is a member of the fuzzy set to some grade, may be even zero.The function that ties a member to each element x of the universe is called the membership function m(x)

Singleton A fuzzy set A is a collection of ordered pairs

A = { x,m(x)}

Item x belongs to the universe and m(x) is grade of membership in A.

A single pair (**x,m(x))** is called **fuzzy singleton**; thus the whole set can be viewed as the union of its constituent singletons.

2.2 Linguistic variables

Just like an algebraic variable takes number as values a linguistic variable takes words or sentences as values.

In this way, a **fuzzy set F** in a universe of discourse **U** is characterized by memberships

mF(u) э (0,1)

in the fuzzy set F. Note that a classical set A in U is a special case of fuzzy set with all membership values mA(u) э (0.1)

The basic concept underlying fuzzy logic is the linguistic variable. A linguistic variable is characterized by a quintuple (**x,T(x), U, G, M)** in *which x is the name of linguistic variable; T(x) is the term set of x, that is, the set of names of linguistic values of x; and M is a semantic rule for associating with each value its meaning.*

Example.

Let us consider the linguistic variable *Symptom* 1. Its term set *S1(symptom 1*) could be: increasing, stationary decreasing, where each term is characterized by a fuzzy set in a universe of discourse *U (-40%, 40%).* We might interpret *decreasing* evolution in percentage between *-40% and –20%, an stationary as evolution between –20% and + 20% and increasing between 20% and 40%.* This terms can be characterized as fuzzy sets whose membership functions are shown in fig 1. For example if the symptom evolution is –25% then the membership degree to the fuzzy subset stationary is equal to zero.

Fuzzy logic provides operations , which acts on fuzzy sets. For example the union A U B of two fuzzy sets is defined as

m(A ∪ B)(x)) = max (mA(x), mB(x)) for all x in U

2.3 Continous and discret representations*.*

There are two alternative ways to represent a membership functions in a computer; continuous or discrete.In the continuous form the membership function is a mathematical function, possible a program. A membership function is for example a triangular.

In the discrete form the membership function and the universe are discret points in a list. Sometimes it can be more convenient with a sampled (discret) representation.

2.4 Fuzzy inference

The process of converting the crisp input data to a fuzzy set A is called fuzzyfication. It maps the input data into their membership functions.

A fuzzy implication is viewed as describing fuzzy relation between fuzzy sets forming the implication.

A fuzzy rule, such *as*

"IF X IS A THEN Y IS B "

is a fuzzy implication which has a membership function $m(A. \rightarrow B)(x, y) \in (0, 1)$

Note that $m(A \rightarrow B)(x,y)$ measures the degree of truth of the implication relation between x and y. The IF part of an implication is called the antecedent (premise) whereas the THEN part is called the consequent.

Using the minimum implication, the membership function of the fuzzy implication is defined as:

$$m(A. \rightarrow B)(x,y) = min (mA(x), mB(y)).$$

Let us consider the following rule template, where X, Y and Z are linguistic variables defined on the universe of discourse U, V and W respectively

.

R) *IF X is A and Y is B then Z is C* *i =1..n*

Given the crisp input (x, y) the goal is to determine the output "Z is C" using fuzzy inference. The most commonly used fuzzy inference methods are so called Max-Min or Max-Product methods which will be described in (6)

The result of the fuzzy inference system is a fuzzy set. The Defuzzification step produce a representative crisp value as the final output of the system. There are several Defuzzification methods. The most commonly used is the Centroid (Center of gravity) defuzzyfier, which provides a crisp value in the center of gravity of the result (the output fuzzy set)

3 Expert system Object

To understand better the application of fuzzy decision making in medicine, in the following paragraphs we describe one method to establish the optimal dosage in the application *of* one medical treatment.

***The method description*.**

The first step in the process of one Expert System development is the establishment of System's object. For the illustration of the medical fuzzy programs we propose an Expert Program (designed to act as an expert for solving a sub-problem from the real world) to assist the physician in the establishment of the dosage of a medical treatment. For a better understanding the Program elaboration process the problem will be much simplified and generalized. We suppose that in the evolution of the patient disease we watch over two symptoms S1 and S2 and it is in application one treatment T. From the previous analysis we know the Symptoms evolution and we wish to establish the next treatment dosage that prove to be necessary.

The S1 and the S2 symptoms are characterized The implementation may prove to be simplistic for some expert systems, however it does illustrate the process. Additional degrees of Symptom *S1 or S2* may be included if needed for the desired system response. This will increase the rule base size and complexity but may also increase the quality of the decision. through percent changes and are labeled as *:*

P (positive) which characterizes an increasing evolution
N (negative) which characterizes an decreasing evolution
Z (zero)which characterizes an stationary state

The notations for the treatment dosage are :

D denote an decreasing in dosage
M denote an maintaining in dosage
I denote an increasing in dosage.

Every symptom or treatment state is evaluated in percentage changes in comparison with the initial moment chosen for comparison.
The following domains (universes of discourse) are established for the symptoms S1 , S2 and the treatment T*:*

For S1 the domain is -40%…….+40%
For S2 the domain is -20%…….+20% (1)
For T1 the domain is -100%…….+100%

4 The Rules establishment

Linguistic rules describing the expert system consist of two parts: an antecedent block (between the **IF** and **THEN** keywords), and a consequent block (following **THEN**).
Depending on the system, it would not be necessary to evaluate every possible input combination (3 by 3) since some outcomes may overlap or never occur.
By making this type of evaluation, usually done by an experimented expert in domain, fewer rules can be evaluated, thus simplifying the processing logic. The rules use the input membership values as weighting factors to determine their influence of the fuzzy output sets on the final output conclusion (in our case the treatment)
Once the functions are inferred , scaled and combined, they are defuzzyfied into a crisp output which represents the decision.

The production rules written in quasi-natural language consist of two parts:
The antecedent part which is content between
IF……………..AND …………THEN
And the consequent part which follows after
………THEN……………
Depending of the elaborating system, it is possible not to evaluate all the possible input combinations.
For example, for a matrix of 5 x 5 (assuming that exist 5 symptoms with 5 states) (linguistic variables) it is possible to be not necessary to evaluate all the 25 possible combinations. A part may seldom appearing and another part not at all appearing.
When using a evaluation procedure which usually is established by the domain expert it will be enough to evaluate a relative few number of rules. In this case the process will be simplified without decreasing the decision precision.
The general structure of Fuzzy Program Expert will be of the type (in quasi-natural language*):*

IF the evolution of the Symptom S1 is decreasing (N) AND
IF the evolution of the Symptom S is increasing (P) THEN
The Treatment T will be increased (I)

Using this writing form for the two Symptoms S1 and S2 and the Treatment T, the following rules structure is established:

Antecedent Block	*Consequent Block*	
IF S1 IS N AND S2 IS N	*THEN T IS D*	
IF S1 IS Z AND S2 IS N	*THEN T IS I*	
IF S1 IS P AND S2 IS N	*THEN T IS I*	
IF S1 IS N AND S2 IS Z	*THEN T IS D*	
IF S1 IS Z AND S2 IS Z	*THEN T IS M*	
IF S1 IS P AND S2 IS Z	*THEN T IS I*	
IF S1 IS N AND S2 IS P	*THEN T IS D*	
IF S1 IS Z AND S2 IS P	*THEN T IS D*	*(2)*
IF S1 IS P AND S2 IS P	*THEN T IS I*	

Writing in matrix form this is *:*

	S1n	S1z	S1p
S2n	Td	Ti	Ti
S2z	Td	Tm	Ti
S2p	Td	Td	Ti

5 The Membership functions

After the rules elaboration the following step is the use of these rules. For this purpose it is necessary to know the membership functions.

The membership function is a graphical or analytical representation of the participation amount for of each input. It associates a weighting with each of the inputs that are processed, - in our case the S1 and S2 Symptoms defines functional overlap between inputs, and ultimately determines an output response, actually the Treatment.

There are different membership functions associated with each input and output response. Some features are *:*

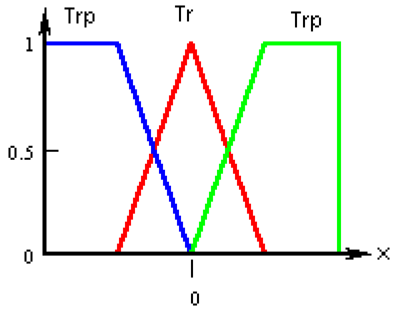
Shape : triangular, trapezoidal standard (using 2nd order polynoms) exponential have been used

Width (of the base of the function)

Center points (center of the member function shape)

Overlap (N&Z, Z&P typically about 50% of with, but it can be less)

Fig1 illustrates the features of the triangular and trapezoidal functions .



Tr ….triangular

Trp….trpezoidal

Fig1 General form for triangular and trapezoidal membership functions

There are many types of membership functions built up either of lines segments (triangular or trapezoidal) or the type sigmoid (polynomial order 2) known as Standard (S or Z functions)

In the first example we have chosen for the *Symptoms S1, S2 and the Treatment T*, the membership functions (msf) of the following type (Fig 2)

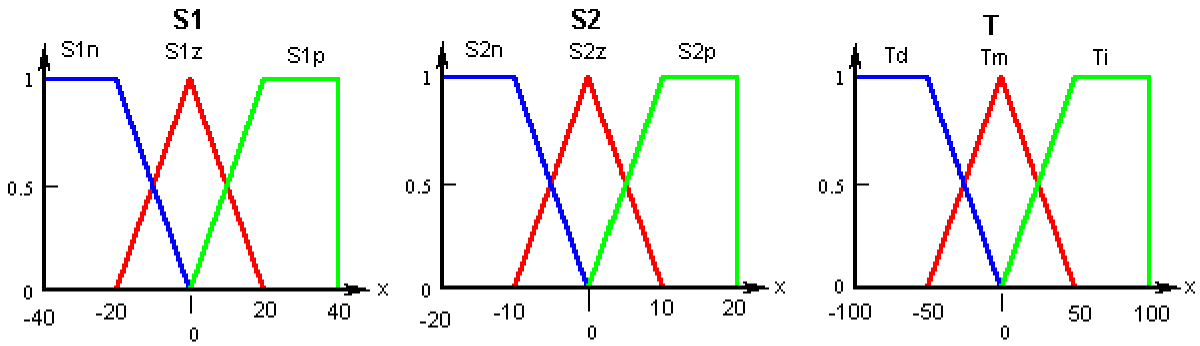


Fig2 *Symptoms S1, S2 and the Treatment T*, the membership functions (msf)

For a fixed x point on the abscise it is possible to result more that one grades of membership functions.

The degree of membership (DOM) is determined by plugging the selected input parameter (S1 or S2) into the horizontal axis and projecting vertically to the upper boundary of the membership functions.
The degree of membership for S1= -10 of (fig2) projects up to the middle of the overlapping part of the “N (decreasing)“ and “Z (maintaining)” function, therefore the result is “N membership = 0.5 and Z membership = 0.5”
Only rules associated with “N” AND “Z” will apply to the output respon

Analytical describing of the membership functions

In almost all use cases is necessary to know not only the graphical representation of the used membership functions but also their analytical representations. The problem that arises is that the membership functions are defined on limited intervals and on the same interval can coexist more that one function.
For following computing a Mathcad Program will be used and, therefore, to describe one function defined on a limited interval we use the step function Φ(x) defined as is illustrated in fig 3 :

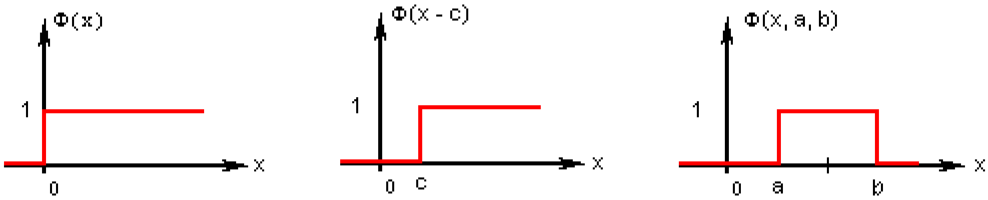


Fig3 Step functions a) *Φ(x)* *b) Φ(x -c)* *c) Φ(x-a)- Φ(x-b)*

The definition of Φ(x) function is

Φ(x) = 1 for x ≥ 0 (3.a)
0 for x < 0

For a translated step function (fig 3) :

Φ (x-a) = 1 for x ≥ a (3.b)
0 for x < a

In fig 3c is represented a function which is defined on the interval a ≤ x ≤ b
To indicate that a function f(x) exists only on the interval x ≥ a we multiply this function with Φ (x-a)
To define a function given only on the limited interval (a, b) we have to write :

[Φ(x-a)- Φ (x-b)] (3.c)

The general form for the membership functions of the Symptoms S1, S2 and Treatment T is illustrated on fig 4. The values of the parameters a, b, c, d and e in each case are:

For Symptom S1 a = -40 ; b = -20; c= 0; d = 20 ; e = 40
For Symptom S2 a = -20 ; b = -10; c= 0; d = 10 ; e = 20
For Treatment T a = -100 ; b = -50; c= 0; d = 50 ; e = 100 *(4)*

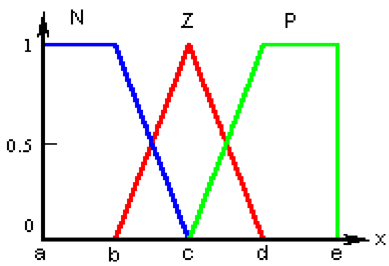


Fig4 General form of S1, S2 and T

For the membership functions that represent the linguistic values of the symptoms S1 and S2 will be used the notations :

S1n(x) for the decreasing membership function of S1

S1z(x) for the maintaining membership function of S1
S1p(x) for the increasing membership function of S1
And, respectively, for the membership of the Symptoms S2
S2n(x); S2z(x); and S2p(x)
For the treatment T we use the notations :
Td for the decreasing membership functions
Tm for the decreasing membership functions
Ti for the decreasing membership functions.
If we introduce the new function :

$$\Phi(x, x1, x2) = \Phi(x-x1) - \Phi(x-x2) \qquad (5)$$

and we take into account the linear functions (Annex 1) that represent the increasing and decreasing membership functions we obtain :

$$S1n(x) = \Phi(x, a, b) + \Phi(x, b, c).(1- \frac{x-c}{c-b}) \qquad a)$$

$$S1z(x) = \Phi(x, b, c).(x-c)/(d-c) + \Phi(x, c, d).(1- \frac{x-c}{d-c}) \qquad b)$$

$$S1p(x) = \Phi(x, c, d).(x-c)/(d-c) + \Phi(x, d, e) \qquad c) \qquad (6)$$

.
The parameter values a, b, c, d was indicated for each case in (4)
The following abbreviations are used :
n *for decreasing*
z *for maintaining*
p *for increasing*

S1n *means decreasing of S1 symptom,* ***S1z*** *maintaining of S1 and* ***S1p*** *increasing of S1. Similar meaning have* ***S2n , S2z*** *and* ***S2p***
We are noting with **w** the truth grade (firing strength) of the rules. For example **w(R1**) mean the firing strength of the **R1** rule. There are obtained:
w*(R1), w(R2)……w(R9)*
The indexes **d, m** and **i** mean decreasing, maintaining or increasing in the case of Treatment **T.** Therefore, **T1d** mean the decreasing membership function of treatment .

6. The rules evaluation

After the system collects the input (data of a real case), the base rule is evaluated. The antecedent (**IF X AND Y**) blocks test the inputs and produce conclusions. The consequent (**THEN Z**) blocks of some rules are satisfied while other are not. The conclusions are combined to form logical sums. In the frame of the inference process each response output member function's firing strength (0 to 1) is determined.
Considering again the rules, the plugging-in the membership function weights from above "S1" selects rules 1,2,4,5,7, 8 while "S2 " selects rules 4 through 9.
"**S1**" and "**S2**" for all rules are combined to a logical **AND** that is the minimum of either term. Of the nine rules selected only four (rules 4, 5,7,8) fire or have non-zero results. This leaves output response magnitudes for only "*Decreasing"* and "*Maintaining*" which must be inferred, combined and defuzzified to return the actual crisp output. In the rules list below, the following definitions apply :
If in a Rule two propositions **X** and **Y** have al memberships **m(X**) and **m(Y**) and are connected through the connector **AN**D then the resulted membership *w(R*) is given as

$$w(R) = \min(m(X); m(Y)) \qquad (7)$$

As input data for the Symptoms **S1** and **S2** was chosen :
S1 show decrease a equal to *–10%* and *S2 an increase* equal *to 5%.*
Introducing these values into relation we get :
S1n = 0.5 S1z= 0.5 and
S2z = 0.5 S2p = 0.5 (8)

Now all the rules will be examined and the degree of truth w(R) will be computed. For example the R1 rule:
IF S1 is N and S2 is N Then T is D

This means that

$$w(R1)= S1n \cap S2n = 0.50 \cap 0$$

because *S1n = 0.5 and S2n = 0*

The use of the **AND** connector gives :

$$min(0.5;0) = 0$$

Using the same reasoning for all rules we get :

Table 1

R2) IF S1 is Z AND S2 is N THEN

$W(R2)= 0.5 \cap 0 = min (0.5; 0) = 0$

R3) IF S1 is P AND S2 is N THEN

$W(R3)= 0. \cap 0 = min (0; 0) = 0$

R4) IF S1 is N AND S2 is Z THEN

$W(R4)= 0.5 \cap 0.5 = min (0.5; 0.5) = 0.5$

R5) IF S1 is Z AND S2 is Z THEN

$W(R5)= 0.5 \cap 0.5 = min (0.5; 0.5) = 0.5$

R6) *IF S1 is P AND S2 is Z THEN*

$W(R6)= 0.5 \cap 0.5 = min (0.5; 0.5) = 0.5$

R7) IF S1 is N AND S2 is P THEN

$W(R7)= 0.5 \cap 0.5 = min (0.5; 0.5) = 0.5$

R8) IF S1 is P AND S2 is P THEN

$W(R8)= 0.0 \cap 0.5 = min (0; 0.5) = 0$

R9) IF S1 is Z AND S2 is P THEN

$W(R9)= 0.5 \cap 0 = min (0.5; 0) = 0$ (8)

It results that the rules **R4, R5, R7** and **R8** will be evaluated (fired) and all have *w (R) =0..5* and the rules **R1, R2, R3** and **R6** will not be evaluated (fired) because their strength w is equal to zero.

Of the 9 rules, only four (*rules 4,5,7, 8*) fire or have non-zero results. The rules *1,2,3,6 and 9* did not fire (the firing strength is zero). This leaves fuzzy output response magnitude for only "**Decreasing**" and "**Maintaining**", which must be inferred, combined, and defuzzified , to return the actual crisp output.

6.1 Inference .

Inference means the evaluation of a implication as :

IF (premise) **THEN** (consequent)

If the rule has more premises connected by **AND** or **OR** connectors then we use for **AND** computing the operator **MIN** and **MAX** operator for **OR** connector.

In the table 1 we indicate the use of AND operator and in the table ..

the use of both operators (**AND** and **OR**)

The computing results are membership degrees and was noted as

w(R1), w(R2)…W(R9) where *R1, R2 ..R9* are the established rules. If in the rule is only on premise then the rule firing strength w(R) is equal to the membership degree. For example

R) IF S1p THEN Td

for *S1p = 0.7* results *w(R) = 0.7*

If two or more rules are referring to the same output membership function then exist more correlation methods

6.2 Correlation Methods*.*

There are several methods of restricting the height of the consequence fuzzy sets.

Correlation Minimum*.*

The most common method of correlation the consequent with the premise truth truncates the consequent fuzzy region at the truth of the premise.

This is called correlation minimum, since the function sets is minimized by truncating it at a minimum of the predicate truth.

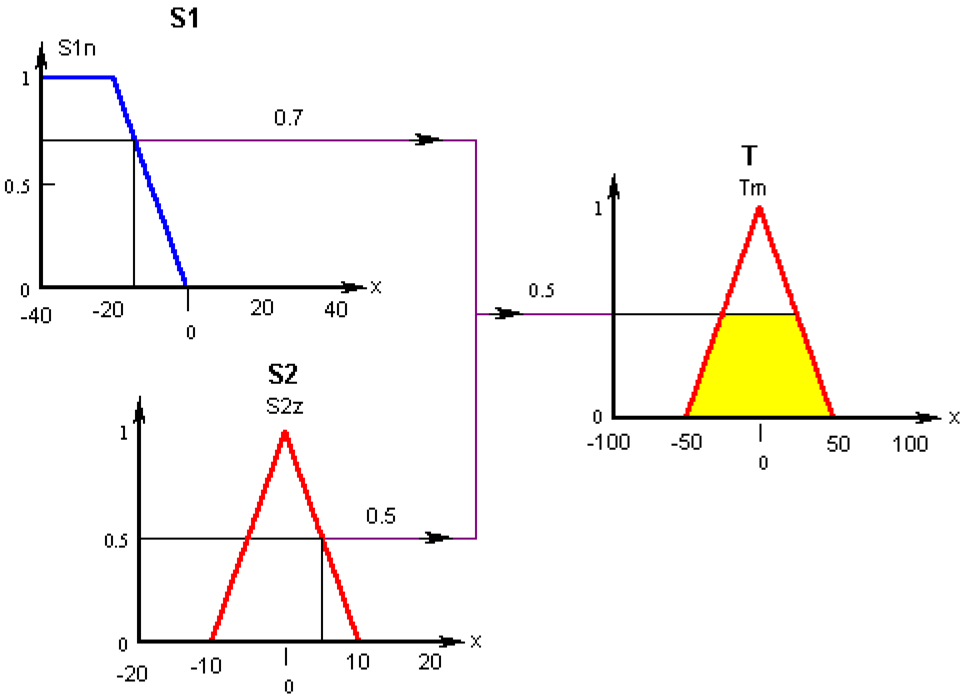


Fig 5 Correlation Minimum of Treatment decision

In fig 5 the evaluation of the treatment is adjusted depending the states of symptoms *S1* and *S2.* At a given moment the membership degree of *S1z (stationary*) is equal to *0.7* and of *S2p (increasing*) is equal to *0.5.*

The fuzzy implication function requires the *minimum* of this two values (*0.5)*

We then use the correlation minimum to truncate the Treatment maintaining fuzzy set at 0.5 level This became the current fuzzy set for Treatment decision...

. In fig 5 it is the result of applying the rule :

IF *Symptom S1 is stationary (S1z)* ***AND*** *Symptom S2 is increasing (S2p)*
THEN *the Treatment is maintaining (Tm) (9)*

Correlation Product

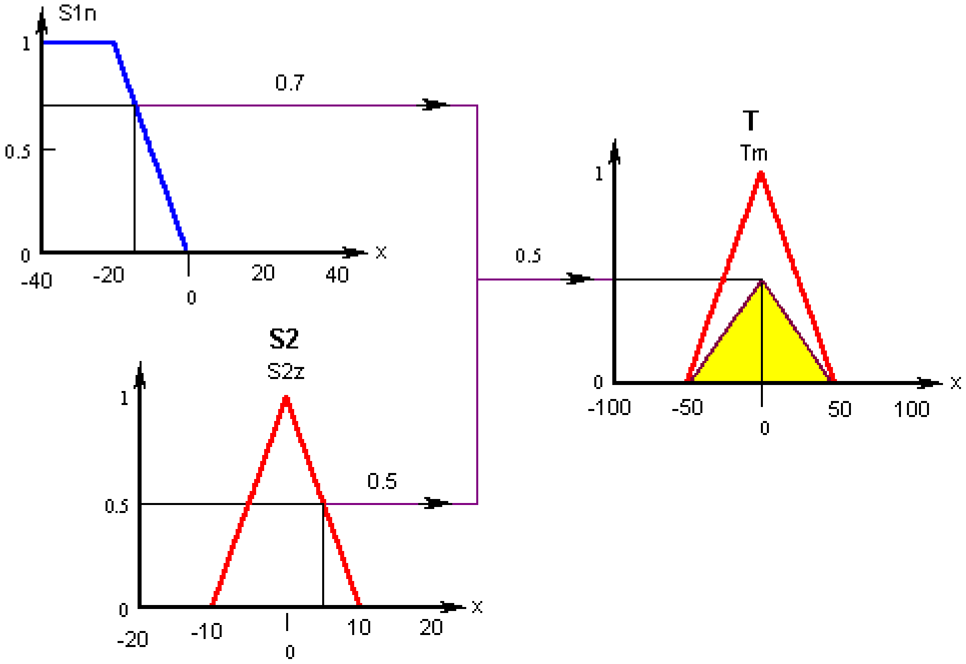
While correlation minimum is the most frequently used technique, correlation product offer an alternative and in many ways , better method of achieving this goal.

With correlation product, the intermediate fuzzy region is scaled instead of truncated.

This has the effect of schranking the original shape of the fuzzy set. In fig 5 it is the result of applying the rule (9)

At a given time *S1 has a 0.7* degree of membership *S1z equal to 0.7and S2 a degree of S2p equal to 0.5*

The minimax implication function requires that we taken minimum of these two values (0.5).We take use the correlation product to scale the Treatment Tm fuzzy set at this 0.5



level. This became the current output fuzzy set for the decision
Fig 6 Correlation Product of Treatment

The ***ROOT-SUM-SQUARE (RSS***) method combine the effects of all applicable rules, scales the function at their respective magnitude, and computes the fuzzy "centroid" of the composite area. In our example was selected since it seemed to give the best weighted influence to all firing rules.

RSS method.
If the fired rules are
R1, R2,…..Rn
With *w(R1), w(R2),….W(Rn)* firing strengths the combined effect of these rules noted as Sq in RSS method is :

$$\mathbf{Sq = [w(R1)^2 + w(R2)^2 + \quad w(Rn)^2]^{0.5}} \qquad (9)$$

In our case we have the following combined effects :
Sqd ….for rules which are referring to the **Td** (*Decreasin*g) membership functions of the treatment T
Sqm ….for rules which are referring to the **Tm** (*Maintaining*) membership functions of the treatment T
Sqi ….for rules which are referring to the **Ti** (*Increasin*g) membership functions of the treatment T

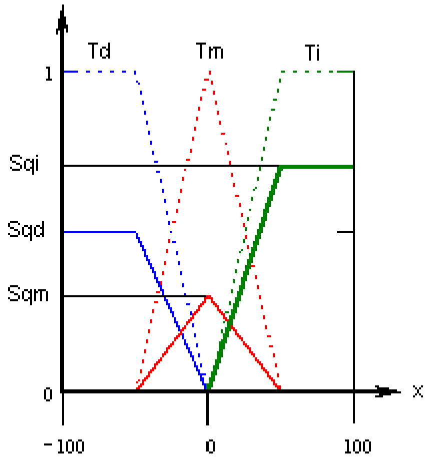


fig 7 Ilustration of RSS method

Referring back to the rules we would notice:

Symptom S1 selects rules 1, 2, 4, 5, 7, 8

Symptom S2 selects rules 4, 5, 6, 7, 8, 9

After combining by **AND** operator (find minimum) for all nine rules only rules 4,5,7,8 fire , or have non zero results. This leaves fuzzy output response magnitude for only “***Decreasing***” and “***Maintaining***“, which must be inferred combined and defuzzifyied to return the actual crisp output.

In conclusion:

The inputs are combined logically using AND operator to produce output response values for the expected inputs. The active conclusions are then combined into logical sum for each output membership function. A firing strength for each output membership function is computed. The logical sums are combined in defuzzifycation process to produce the crisp value

7. Defuzzification

The Defuzzification step is one of the most important steps to be considered in the process of designing an Expert System, because this establish the finally outcome.

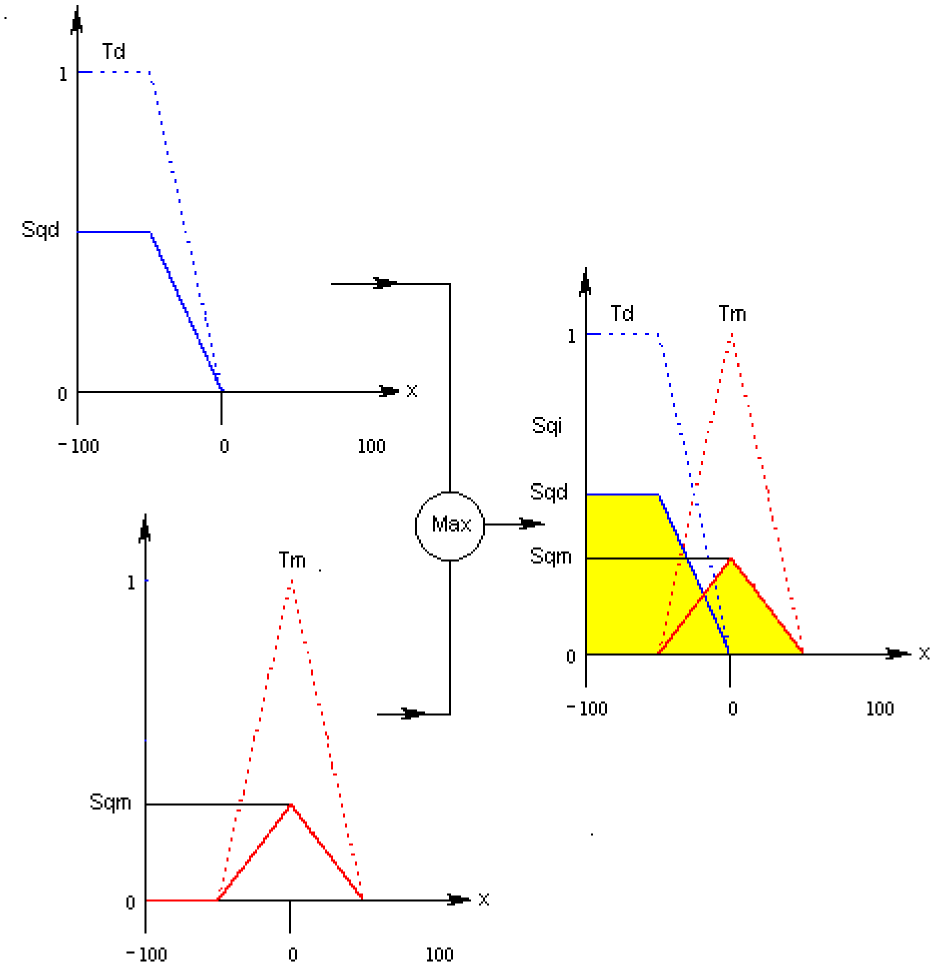
To find the actual value of the corresponding scalar d (that represents the value of the treatment adjusting (dosage) we must find a value that best represents the information contained in the fuzzy set T. This process is illustrated in fig 8 called Defuzzification.

Such a process yields the expected value of the variable for a particular execution of the fuzzy model .Defuzzification is the final phase of a fuzzy reasoning

.

In fuzzy models, there are several methods of determining the expected value of the solution fuzzy region. These are the methods of decomposition, also called methods of Defuzzification and describe an expected value for the final fuzzy state space.

Fig 8.Defuzzyfication



In our proceeding the output membership function for every linguistic value of the treatment results trough multiplication of the respectively Sq value (Sq for Decreasing, Sqm for Maintaining, and Sqi for increasing) with their chosen membership functions (trapezoidal for Decreasing and Increasing and triangular for Maintaining.

The resulted membership function surfaces are summated and results the final total surface of the membership output function.

In our case every output membership function center value is multiplied by the Sq value. All the obtained terms are summated and the obtained value is divided by the sum of the Sq factors. For the output abscissa value we get :

$$\mathbf{X_{cnteer}} = \frac{TdCenter * Sqd + Tmcenter * Sqm + TiCenter * Sqi}{Td + Tm + Ti} =$$

$$\frac{-100 * 0.866 + 0.5 * 0.0 + 100 * 0.0}{0.866 + 0.5 + 0.0.} = -63.4 \qquad (10)$$

This mean that the Treatment dosage is to be decreased with 63.4%.

For the ongoing example, as center for the output membership function was chosen
The range limit for trapeze (x = -100 and x = 100)
The base of the middle for triangle (x = 0)

This option is doubtful and therefore another Defuzzification method will be also analyzed.
The used Defuzzification method is known as **Max-Product** because every output membership function results as product between the firing strength **Sq** and the *membershi*p

function. The total output membership function is obtained by summing the resulted membership functions (*Treatment Decreasing, Maintaining , Increasing*)

Comparative Analysis.

To illustrate the influence of the different input data (the *S1 and S2 Symptoms* values) and different Defuzzification methods we selected 4 cases with the following inputs data :

Case 1	Case 2	Case 3	Case 4
X1 = -10	x1 = -15	X1 = 15	x1= -5
X2 =5	x2 =3	X2 =7	x2 = 3

Because of the relatively large amount of numerical computing required, the Mathcad program is used.

The membership functions analytic forms for *S1 and S2 symptoms* and *treatment T* are indicated in (4). Since only four rules (R1,R4,R7 and R9) referring to the Td (treatment) are fired (w(R) >0) using (9) we get :

$$Sqd = [\ w(R1)^2 + w(R4)^2 + w(R7)^2 + w(R8)^2\ ]^{0.5}$$

For Tm only one rule is referring (R5) and therefore :

$$Sqm = [\ w(R5)^2]^{0.5} = w\ (R5)$$

For Ti no one rule is referring and therefore :

$Sqi = 0$

To compute the membership functions we used the relations (6) and we get :

Case 1		Case 2		Case 3		Case 4	
X1 =-10	x2= 5	X1 =-15	x2=3	x1 =15	x2= 7	x1 =- 5	X2= 3
S1n= 0.50	S2n= 0	S1n=0.75	S2n= 0	S1n= 0	S2n=0	S1n=0.25	S2n=0
S1z= 0.50	S2z=0.50	S1z=0.25	S2z=0.70	S1z=0.25	S2z=0.30	S1z=0.75	S2z=0.70
S1p= 0	S2p= 0.50	S1p= 0	S2p = 0.30	S1p= 0.75	S2p = 0.70	S1p= 0	S2p = 0.30

For the firing strength of the rules results :

$$
\begin{aligned}
w(R1) &= S1n \cap S2n \\
w(R2) &= S1z \cap S2n \\
w(R3) &= S1p \cap S2n \\
w(R4) &= S1n \cap S2n \\
w(R5) &= S1z \cap S2z \\
w(R6) &= S1p \cap S2z \\
w(R7) &= S1n \cap S2p \\
w(R8) &= S1z \cap S2p \\
w(R9) &= S1p \cap S2p
\end{aligned}
\qquad (11)
$$

Introducing the numerical values of *S1n, S1z.S1p, S2n,S2z, S2p* for every case we get the result given in table :

	Case 1		Case 2		Case 3		Case 4	
w(R1)	0.50 ∩ 0.00	0.00	0.75 ∩ 0.00	0.00	0.00 ∩ 0.00	0.00	0.25 ∩ 0.00	0.00
w(R2)	0.50 ∩ 0.00	0.00	0.25 ∩ 0.00	0.00	0.25 ∩ 0.00	0.00	0.75 ∩ 0.00	0.00
w(R3)	0.00 ∩ 0.00	0.00	0.00 ∩ 0.00	0.00	0.75 ∩ 0.00	0.00	0.00 ∩ 0.00	0.00
w(R4)	0.50 ∩ 0.50	0.50	0.75 ∩ 0.70	0.70	0.00 ∩ 0.00	0.00	0.25 ∩ 0.70	0.25
w(R5)	0.50 ∩ 0.50	0.50	0.25 ∩ 0.70	0.25	0.25 ∩ 0.25	0.25	0.75 ∩ 0.70	0.70
w(R6)	0.00 ∩ 0.50	0.00	0.00 ∩ 0.70	0.00	0.75 ∩ 0.30	0.30	0.00 ∩ 0.70	0.00

w(R7)	0.50 ⋂ 0.50	0.50	0.75 ⋂ 0.30	0.30	0.00 ⋂ 0.00	0.00	0.25 ⋂ 0.30	0.25
w(R8)	0.50 ⋂ 0.50	0.50	0.25 ⋂ 0.30	0.25	0.25 ⋂ 0.25	0.25	0.75 ⋂ 0.30	0.30
w(R9)	0.00 ⋂ 0.50	0.00	0.00 ⋂ 0.30	0.00	0.75 ⋂ 0.70	0.70	0.00 ⋂ 0.30	0.00

Using the relations (9) for *Sqd,Sqm, Sqi* ,for the 4 cases we get :

	Case 1	Case 2	Case 3	Case 4
Sqd	0.77	0.82	0.42	0.46
Sqm	0.50	0.25	0.25	0.70
Sqi	0.00	0.00	0.58	0.00

The crisp values for the abscissa of the output functions surfaces COG (center of gravity) using the relation (12) and for the different 4 cases we get :

	Case 1	Case 2	Case 3	Case 4
Xcg	-60.6	-76.6	12.8	-39.8

This mean that in the **case 1** the treatment dose is to be **decreased with 60.6%, in case 2** to **decrease with 76.6 %** ,in the **case** 3 to increase with **12.8**% and in **case 4** to **decrease** with **39.8 %**

8 Other Defuzzification methods

A more plausible Defuzzification method than that used before is given by the relation :

$$Xc = \frac{\int_a^b x.y(x).dx}{\int_a^b .y(x).dx} \qquad (12)$$

where a and b are the output (treatment) membership function's domain limits (a = -100; b=100) and y(x) is the total output membership function. In our case y(x) is given by the relation :

$$\mathbf{y(x) = [\ Sqd.Td(x) + Sq\ m.Tm(x) + Sqi.Ti(x)]} \qquad (13)$$

where *Td(x), Tm(x) and Ti(x)* are defined by analytical function similar with that used for *S1n, Siz, and S1p, namely :*

$$Td(x) = \Phi(x, a,b) + \Phi(x, b,c).[1-\frac{x-b}{c-b}\ ]$$

$$Tm(x)= \Phi(x, b,c).\ \frac{x-b}{c-b}\ + \Phi(x, c,d).[\ 1-\frac{x-b}{c-b}\ ]$$

$$Ti(x) = \Phi(x, c,d).\frac{x-c}{d-c} + \Phi(x, d,e). \qquad (14)$$

The values used for the parameters are:
a= -100 b= -50 c= 0 d= 50 e=100

Using the relations (12) and (14) we get for the abscissa (Xc) of the total output membership function center of gravity ;

	Case 1	Case 2	Case 3	Case 4
Xcg	-42.6	-50.8	26.7	-30.5

This method considered as the most plausible Defuzzification method has the disadvantage of needing a large amount of numerical computing.

8.1 Mean of maxima Defuzzification method (MOM)

Now we describe briefly the Defuzzification method Mean of Maximum (MOM) in the case of Max-Product inference type. The total output membership function is of the form (fig 9)

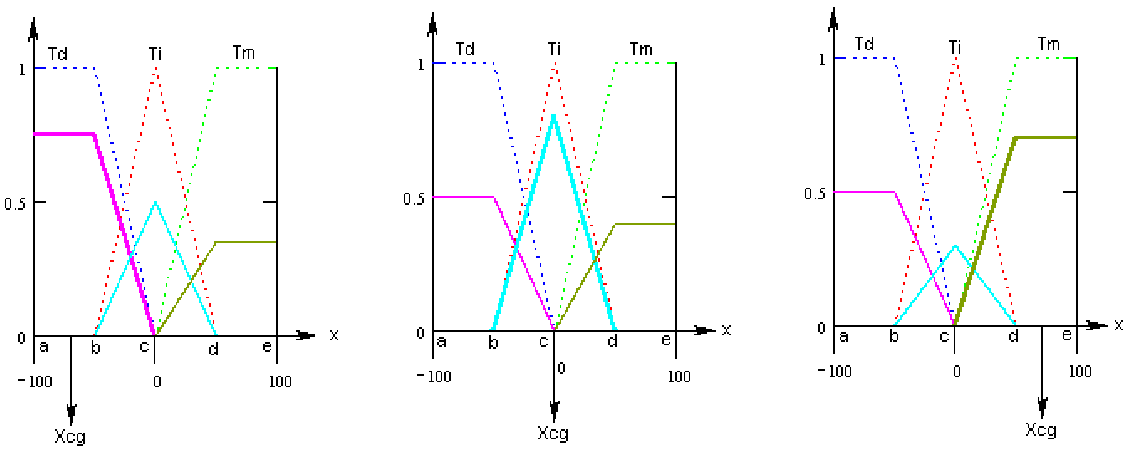


Fig 9 The total output membership function

If we compare the 3 obtained values *Sqd, Sqz and Sqp* we distinguish 3 different possible cases.

a- Sqd has a maximum value.

In this case the domain in which the membership function Td is maxim is a—b. The value of the center of this interval is (a+b)/2. Therefore the expected decision point is

Xc = (a+b)/2

b-The Sqi has a maximum value.

By analogy, in this case

Xc = (d+e)/2

c- Sqm has a maximum value.

In this case there is not an interval when the membership function Ti has a maximum but only one point namely the center c=0. In this case

Xc = c = 0

We do not recommend the Defuzzifications First of Maxima and Last of Maxima in applications as for the cases presented here.

8.2 Defuzzification methods using discrete data

The principle of this method consist in the fact that instead of computing of the output function center of gravity, by using the relation (8) based on integral, we use the summing of the output membership function values computed in relatively reduced number of points.

We note **$y(x_j)$** the output membership function calculated at the discrete number of points x_j, where "j" is a relatively small number. Usually the x_i points are chosen and disposed at equal intervals on the axis that represent the membership functions domain of definition.

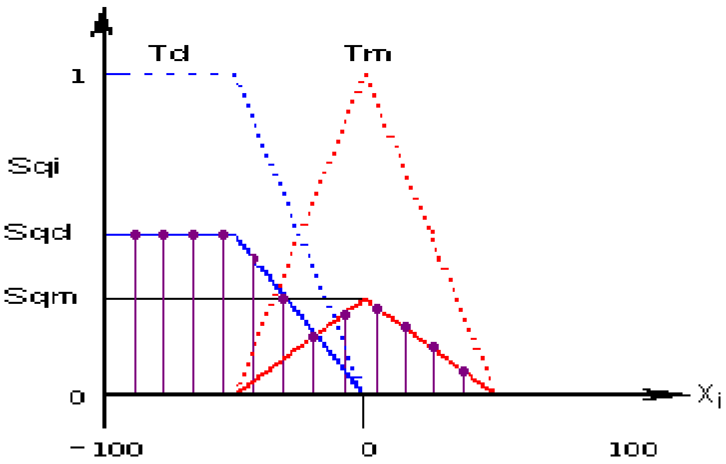


Fig 10 Treatment msf defuzzification using discrete data

The expected value, namely the value of abscissa of the total output membership function center of gravity point is given by :

$$X_c = \frac{\sum_{i=1}^{n} x_i \cdot y(x_i)}{\sum_{i=1}^{n} x_i \cdot y(x_i)} \qquad (15)$$

In the analyzed case the y(x) function is given by the relation (13) and the x_j points were chosen :

a) all points are distributed at 10 percent intervals between them, respectively:

x_j = -100,-90,-80 ..90, 100 (20 points altogether)

b) the number of used points is 10 (20 percent interval between them)

c) the number of used points is 40 (5 percent interval between them)

Using the relation (10) we get the following final values of the abscissa points of gravity center of the total output membership function :

	Computed based on relation (8)	Computed on 40 points	Computed on 20 points	Computed on 10 points
Case 1	-42.6	-43.9	-45.3	-46.5
Case 2	-50.8	-52.6	-53.5	-56
Case 3	8.4	5.6	2.7	3.1
Case 4	-30.3	-31.1	-32.7	-34.

Conclusion:

Even we consider that the Defuzzification method for computing the abscissa value of the center of gravity of total output function based on integral (relation 12) as the most plausible, we recommend the use of the centroid Defuzzification method relying on discrete points. When we are examining the output data given in Table 2?, we notice the difference between these data is not too important. The increasing of the discrete number of employed points is to be examined in every specific case and to be maintained moderate,

8.3 Rules with combined AND and OR operators use.
To diversify the application illustrations, the following the data of the case 2 are used with the difference that inside of the rules antecedents are present not only AND operators but **OR** too.
It results that the input data and the computed membership values are :

Case 2

X_1 = -15	X_2 = 3
S1n = 0.75	S2n = 0
S1z = 0.25	S2z = 0.7
S1p = 0	S2p = 0.3

The rules are :

R1) IF S1n OR S2n THEN Td
R2) IF S1n AND S2z THEN Ti
R3) IF S1n AND S2p THEN Ti
R4) IF S1z OR S2n THEN Tm
R5) IF S1z AND S2z THEN Tm
R6) IF S1z OR S2 THEN Tm
R7) IF S1p AND S2n THEN Td
R8) IF S1p OR S2z THEN Td
R9) IF S1p OR S2p THEN T (16)

The rules *R1, R7, R8* are referring to the output membership function *Td (Treatment decreasing*), the rules *R4, R5, R6* are referring to the membership function *Tm (Treatment maintaining*) and the *rules R2, R3, R9* are referring *to Ti (Treatment increasing*)
Using the **Max-Product** inference method for the output membership function strength we get :

w(R1) = max (S1n; S2n) = max (0.75; 0.00) = 0.75
w(R2) = min (S1n; S2z) = min (0.75; 0.70) = 0.70
w(R3) = min (S1n; S2p) = min (0.75; 0.30) = 0.30
w(R4) = max (S1z; S2n) = max (0.25; 0.00) = 0.25
w(R5) = min (S1z; S2z) = min (0.25; 0.70) = 0.25
w(R6) = max (S1z; S2p) = max (0.25; 0.30) = 0.30
w(R7) = min (S1p; S2n) = min (0.00; 0.00) = 0.00
w(R8) = max (S1p; S2z) = max (0.00; 0.70) = 0.70
w(R9) = min (S1p; S2p) = max (0.00; 0.30) = 0.30 (17)

For the output membership firing strength noted in this case with Sd (Decreasing), Sm (Maintaining) and Si (Increasing) we get :

Sd = max [w(R1); w(R7);w(R8)] = max (0.75;0.00;0.70) = 0.75
Sm = max [w(R4); w(R5);w(R6)] = max (0.25;0.25;0.30) = 0.30
Si = max [w(R2); w(R3);w(R9)] = max (0.70;0.30;0.30) = 0.70 (18)

For the total output membership function results :

y(x) = Sd.Td(x) + Sm.Tm(x) + Si.Ti(x) = 0.75.Td(x)+ 0.30.Tm(x)+ 0.7.Ti(x)

Td(x), Tm(x) and Ti(x) are functions given by (14)
For comparison, the total output function Defuzzification y(x), is accomplished in two variants.
The first one uses for the abscissa of output membership function the gravity center the integral (12) and second variant uses the discrete representation of the total membership

function representation $y(x_j)$ and the relation (15) In this variant five cases are analyzed in each case the number of discrete point is different, so :

n = 200 ; 100; 50 ; 20 ; 10

In the variant in we use the integral to determine the abscissa value of the output membership function center of gravity **Xcg** results

$$\mathbf{Xcg} = \frac{\int_{-100}^{100} x.y(x)dx}{\int_{-100}^{100} y(x).dx} = -48.3$$

In the variant of discret defuzzification method the output membership function center of gravity Xcd is given by :

$$\mathbf{X_c} = \frac{\sum_{i=1}^{n} x_i . y(x_i)}{\sum_{i=1}^{n} x_i . y(x_i)}$$

For the values of different number of discrete points “j” result :

N	200	100	50	20	10
Xcd	-48.5	-48.8	-49.2	-51	-53.4

Conclusion:

If we again consider that the method of Defuzzification using integral operation (rel 12) is the most plausible, it results that the use of Defuzzification in which the output membership function is computed by discrete points give also acceptable precision.

As the data given in the above table show, the precision diminish if the number of computing discrete points diminishes also. However the obtained output data for Xcd are acceptable if we take into account that is a matter of approximate reasoning.

Similarly with the precedent case it is recommended to use the discrete method of Defuzzification using a reasonable number of discrete points.

9 Comparison of the decision obtained using different membership functions types

In the Fuzzy Expert Systems a doubtful problem is represented by the establishment of the linguistic variable membership functions used both for antecedent of rules and for output function too.

The most used functions are triangular or trapezoidal types. In the Annex A there are given the analytical and graphical representation of the characteristics of these functions.

Beside these types many other types of functions are used. In the next analyze we use the Standard functions of type *S* and *Z* (illustrated in Annex A). The membership functions of these type, in our case are :

(We abreviated the Zadeh’s Standard functions with italic ***S*** *to make difference from the abreviations of Symtoms* S1 *and* S2)

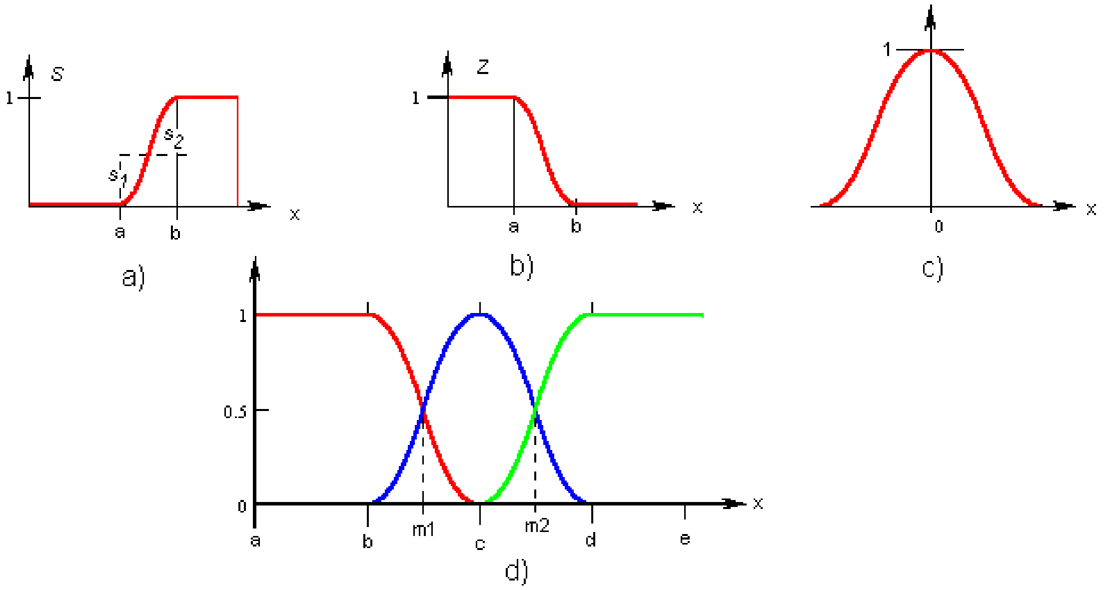


Fig 11 Standard type membership functions

As we already mentioned a difficulty in describing these functions in analytical form is given by the fact they are defined on limited intervals. For computing we use again the Mathcad Program and the introduced step function $\Phi(x, a,b) = \Phi(x-a) - \Phi(x-b)$.
To describe the membership functions Sn(x),Sz(x) Sp(x) and Td(x), Tm(x) and TI(x) in the case of Standard functions use, we define these functions using the following aiding functions:

$$s1(x,a,c) = 2\left(\frac{x-a}{c-a}\right)^2$$

$$s2(x,a,c) = 1 - 2\cdot\left(\frac{c-x}{c-a}\right)^2 \qquad (19)$$

where a and c are parameter indicated on fig 11
$\Phi(x,b,m1)$
Using these functions and taking into account the fact that both curves *Sa, Sb* and *Za, Zb* (fig 11) are formed from two different segments, defined on different intervals we get for these functions :

$$Sa(x) = \Phi(x,b,m1).s1(x,b,c) + \Phi(x,m1,c).s2(x,b,c)$$

$$Za(x) = \Phi(x,b,m1).(1-s1(x,b,c)) + \Phi(x,m1,c).(1-s2(x,b,c))$$

$$Sb(x) = \Phi(x,0,m2).s1(x,0,d) + \Phi(x,m2,d).s2(x,0,d)$$

$$Zb(x) = \Phi(x,0,m2).(1-s1(x,0,d) + \Phi(x,m2,d)(1-.s2(x,0,d) \qquad (20)$$

m1 and m2 are the middles of the intervals b..c and c..d
For the expressions of the Fuzzy variables *Sn(x), Sz (x) ,Sp(x) and Td(x),Tm(x) and Ti(x)* it results:

$Sn(x) = \Phi(x,a,b) + Za(x).$
$Sz(x) = Sa(x) + Zb(x).$
$Sp(x) = Sb(x) + \Phi(x,d,e).$
$Td(x) = \Phi(x,a,b) + Za(x).$
$Tm(x) = Sa(x) + Zb(x).$
$Ti(x) = Sb(x) + \Phi(x,d,e).$ (21)

The values of **a, b,c,d,e** parameters are ;

	a	B	C	d	E
Symptom 1	-40	-20	0	20	40
Symptom 2	-20	-10	0	10	20
Traitment	-100	-50	0	50	100

For the decision computing (treatment evolution) in this example we use the AND operator in the antecedent part of the rules and the **RSS** operator to compute the firing strength of the output membership functions.
To obtain the total output membeship function y(x) we use the relation:

y(x) = Sqn.Td(x) + Sqz.Tm(x)+Sqp.Ti(x) (22)

The abscissa of the center of gravity are computed using the integral relation (12)
We want to establish the effect of the Standard membership functions type in comparison with the triangular and trapezoidal type functions.
We analyze again two cases, for a valid comparison. In the first case the *Symptom S1* indicates a *decreasing with 15% (x1 = -15*) and the *Symptom S2 an increasing with 7%* (x2= 7)
In the second case *S1 and S2 symptoms* show an increase *with 7% (x1= x2 =7*).
The values of the membership functions of S1 and S2 are different although x1=x2, because the definition intervals are different.
The first *(S1*) is defined between *–40 % and 40% and S2 between –20% and 20%.*
The obtained results will be compared with the results obtained by using trapezoidal and triangular membership functions.
For this comparison we have to compute the values of the membership functions *S1n, S1z*, *S1p, S2 z n, S2 S2p* and the firing strengths *Sqd, Sqm and Sqi*, and finally the abscissa of the output function center of gravity. These are :

The values of the membership function of Simptoms S1 and S2 in the case 1

x1= 7 x2 =7

	S1n	S1z	S1p	S2n	S2z	S2p
Standard functions	0.00	0.775	0.245	0.00	0.18	0.82
Linear functions	0.00	0.65	0.35	0.00	0.30	0.70

The values of the membership function of *Simptoms S1 and S2* in the case 2

x1= -15 x2 =7

	S1n	S1z	S1p	S2n	S2z	S2p
Standard functions	0.875	0.125	0.00	0.00	0.18	0.82
Linear functions	0.75	0.25	0.25	0.00	0.30	0.70

The values of the firing strength *Sqd, Sqm, Sqi* in the case 1

x1= 7 x2 =7

	Sqd	Sqm	Sqi
Standard functions	0.755	0.18	0.84
Linear functions	0.7	0.3	0.762

The values of the firing *strength Sqd, Sqm, Sqi* in the case 2

x1= -15 x2 =7

	Sqd	Sqm	Sqi
Standard functions	0.849	0.125	0.00

Linear functions	0.78	0.25	0.00

The values of the abscissa of the output membership function center of gravity in the case 1

x1= 7 x2 =7

	Xcd
Standard functions	2.69
Linear functions	2.28

The values of the abscissa of the output membership function center of gravity in the case 2

x1= -15 x2 =7

	Xcd
Standard functions	-56.28
Lineaar functions	-50.35

10. Conclusions

From the comparative analysis of both cases, case 1 and case 2, we notice that neither the values of the membership functions S1n, S1z,S1p, S2n, S2z, S2p nor the firing strength Sqd, Sqm, Sqi in the both types of membership functions (triangular- trapezoidal or standard S or Z) does not differ essentially.

The values of the abscissa of the total output membership function (Center of gravity) that is the most important in the two analyzed cases differ in our example with less then 6 percent.

In conclusion we recommend the use of triangular and trapezoidal membership functions tacking into account that the precision with which we operate does not justify the use of the standard functions which need more complicate computing.

Also we recommend to use the method with discretised data for Defuzzification (using a reasonable number of discrete points)

What is really very important is the correctness of the rules and therefore we have to pay a big attention on their elaboration.

Annex

In the fuzzy logic use a disputable question is the selection of forms for the linguistic variable membership functions. The most used types of membership functions are the linear (triangular and trapezoidal) fig A1

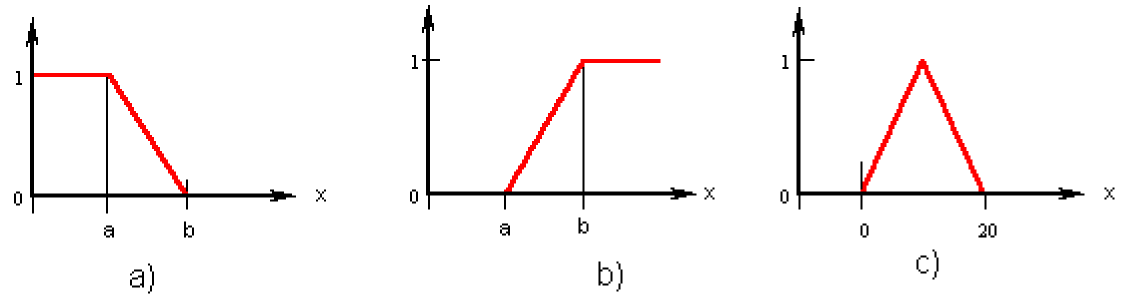


Fig A1 Triangular and trapezoidal m.s.f

The definition relations are :

$$msf1(x) = \begin{cases} 0 & \text{if } \; x< a \\ \dfrac{x-a}{b-a} & \text{if } \; a \le x < b \\ 1 & \text{if } \; x> b \end{cases} \qquad (A1)$$

$$msf2(x) = \begin{cases} 1 & \text{if } \; x< a \\ 1- \dfrac{x-a}{b-a} & \text{if } \; a \le x < b \\ o & \text{if } \; x> b \end{cases} \qquad (A2)$$

Besides the linear functions the Standard functions using expressed by second grade polynoms.

There are also presented in the fig A2. These are known as *S* and *Z* functions.

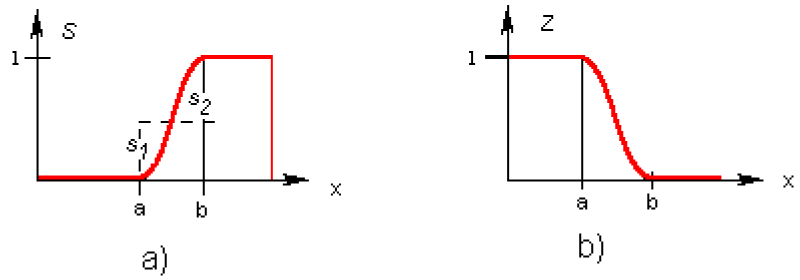


Fig A2 Standard type msf

The definition relations are given by (20):

$$s1(x,a,c\,)= 2\;(\frac{x-a}{c-a})^2$$

$$s2(x,a,c) = 1 - 2\,.\,(\frac{c-x}{c-a})^2$$