Phase dynamics of injection locked spin-torque nano-oscillators: from synchronization to Ising machines

Mateo Ibarra Gomez[1], Arthur Courberand[1], Abderrazak Hakam[1], Chloé Chopin[1],
Philippe Talatchian[1], Ursula Ebels[1] and Liliana D. Buda-Prejbeanu[1*]
[1]Univ. Grenoble Alpes, CEA, CNRS, Grenoble INP, SPINTEC, 38000 Grenoble, France

The phase dynamics of coupled and/or injection locked arrays of spin torque nano-oscillators (STNO) can be harnessed for novel hardware concepts within unconventional computing schemes [1] or for smart communication applications [2]. Here we address the case of an array of weakly coupled STNOs that are simultaneously injection locked to an RF signal whose frequency is close to two times the free running STNO frequency. This *2f* - injection-locking leads to a phase binarization (π-periodic values of the oscillator phase) that can be exploited as a binary spin to implement an Ising machine (IM) for solving combinatorial optimization problems (COP) [3]. For this purpose, the problem is mapped on the Ising Hamiltonian $\mathcal{H}_{Ising} = -\sum_{<i\ j>} J_{ij}\sigma_i\sigma_j - \sum_i h\sigma_i$ that describes a 2D array of binary-valued spins σ_i, σ_j and where the coupling constants J_{ij} encode a given problem. The inherent convergence towards the global energy minimum can then be used as an algorithm to determine the solution of the COP.
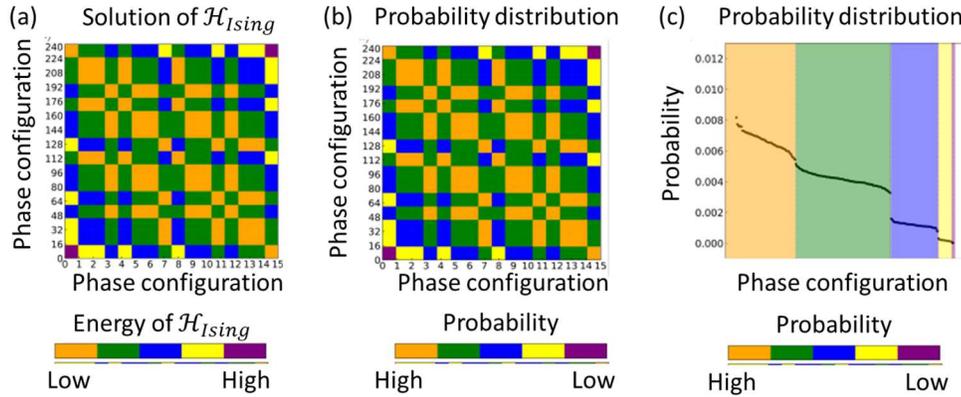

Figure 1 : Comparison of the solution of (a) the Ising Hamiltonian $\mathcal{H}_{Ising}$ for N=8 spins, with 256 highly degenerate spin states, resulting in only 5 different energy levels indicated by the color code and (b) the probability distribution for N=8 coupled identical STNOs for V_{DC} = 1.5 V, rf source frequency 9.618 GHz and a ratio of detuning to locking range δ/ΔΩ = 0.6. The total simulation time is 200 µs and the temperature T=50 K. The probabilities are grouped into five levels according to (c), where the states of highest to lowest probability are plotted.

To explore the stochastic phase dynamics within an Ising system based on in-plane magnetized STNOs the the coupled Landau-Lifshitz-Gilbert-Slonczewski equations including thermal fluctuations have been solved numerically. Combing these numerical simulations with analytical models provided by the non-linear auto-oscillator theory [4] allows one to extract the instantaneous phase of single and *N* coupled STNOs ($N \geq 2$). Due to thermal fluctuations, the phase of each STNO jumps stochastically between its two binary states and, therefore, phase state probabilities and phase correlations between different STNOs are extracted. For a single oscillator, both phase states are equiprobable in the isolated and coupled case. However, the probabilities of the 2^N collective phase states are unequal when the coupling between STNOs is activated. The probability distribution of the phase states of all coupled STNOs is analyzed as a function of the coupling parameters (strength, sign, type of coupling), frequency mismatch and operational point in order to determine the parameter range that provides the solution to an optimisation problem. Results will be illustrated for the MaxCut problem (J_{ij}=K<0) for which the energy levels of the corresponding Ising Hamiltonian $\mathcal{H}_{Ising}$ are determined for all possible phase state combinations (Fig. 1a). The phase states with lowest energy are expected to correspond to the phase states of highest probability (Fig.1b). We first present results for identical STNOs and then address the important question of non-identical STNOs to guide experimental implementations of an Ising machine.

[1] J. Grollier, D. Querlioz, K. Y. Camsari, et al., *Nature Electronics* 3, 360–370 (2020)
[2] A. Litvinenko, P. Sethi, C. Murapaka, et al, *Physical Review Applied* 16 (2021)
[3] T. Wang and J. Roychowdhury, Unconventional Computation and Natural Computation 11493, 232–256 (2019)
[4] A. Slavin and V. Tiberkevich, *IEEE Transactions on Magnetics* 45, 1875–1918 (2009)